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The time constant of double pipe and one pass shell-and-tube heat exchangers in the case of varying fluid flow rates

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Abstract—In this paper we try to characterise the behaviour of a double pipe or shell and tube heat exchanger in transient phase when a sudden change of the flow rate is imposed at the entrance of one of the two inlets. A simplified model is proposed: a two parameter model with a time lag and a time constant. An analytical expression of the time constant is obtained by using the energy balance equation. Experimental investigations show a good agreement with theoretical results. This method can be generalised to describe the response of any variation of the flow rates at the entrance of the heat exchanger.

1. INTRODUCTION

In many industrial applications, heat exchangers such as thermal radiators, solar collectors, classical industrial heat exchangers or complex networks are usually a part of a system. During their normal operation or operating conditions such as time varying inlet temperature or flow rates, they can be submitted to unsteady behaviour. So, an accurate knowledge of the thermal response of such a system during an unsteady period of operation is very important for effective controls, as well as for the understanding of the adverse effects which usually result in reduced thermal performance or increased thermal stresses. In many situations, the results of steady-state are extrapolated to describe the behaviour of the unsteady state, but this method does not suit well in most of the cases, so there is a need to develop models for the unsteady state .

Different methods have been used to study the transient behaviour of heat exchangers. In the case of varying inlet temperature, some authors such as Gilles [1], Correa and Marchetti [2] used a mean or a local heat transfer coefficient deduced from steady-state behaviour. The first author applied this method in the case of double pipe heat exchanger, but the last ones generalised it to the case of shell and tube heat exchangers. In a similar manner, Haddad [3, 4] subdivided the heat exchanger in many cells of width Δx and he considered a constant heat transfer coefficient in every cell. Another kind of method, that avoids the use of this coefficient during the transient phase, is the two-parameter method with time lag and time constant. This method has been used by Gögüs and Ataer [5] in the case of cross-flow circumferentially finned tube heat exchanger and has been performed by Pierson [6] in the case of double pipe heat exchanger when the inlet temperature of one fluid is submitted to a step of temperature. Thereafter, Hadidi [7] has extended this method in the case where the two inlets are simultaneously submitted to steps of temperature.

In the case of variable flow rate, Gilles [1] and Law [8] have related the Laplace transform of the exit temperature to the Laplace transform of the inlet temperature and flow rate by the mean of transfer equations. Correa and Marchetti [2] have subdivided a baffled shell-and-tube heat exchanger in many cells and made the assumption that the heat transfer coefficient is constant in every cell. The validity of these methods is limited to the case of low flow rate. Some computer codes such as Cetuc [9] and Trio-VF [10] have been developed to study the transient behaviour of heat exchangers by using the control volume method. Azilinon [11] used the two-parameter method in the case of double pipe heat exchanger when the inlet flow rate of one fluid is submitted to a step of flow rate. Later Guellal [12] has extended this method in the case where the two inlets are simultaneously submitted to steps of flow rate. Recently, a general formulation of the two-parameter method for both cases when a sudden change of the temperatures or the flow rates is imposed on the two inlets of a double pipe heat exchanger is exposed by the authors in ref. [13].

In this paper, we expose the method of two-parameter in the case of double-pipe heat exchanger for laminar and turbulent flows. This exposure is followed by a parametric study of the influence of some parameters on the time constant and a comparison of the theoretical and experimental results. Then we extend the model to the case of shell-and-tube heat exchanger

NOMENCLATURE

C	heat capacity, [J/K^{-1}]
E	effectiveness, dimensionless
h	convection heat transfer coefficient [W m^{-2} K^{-1}]
k	overall heat transfer coefficient [W m^{-2} K^{-1}]
L	length of the heat exchanger [m]
$\vec{n}$	normal to the surface
NTU	number of transfer units [dimensionless]
q	mass flow rate [kg s^{-1}]
q_t	heat capacity flow rate [W K^{-1}]
r	radius [m]
R	unbalance factor [dimensionless]
Re	Reynolds number [dimensionless]
S	surface [m^2]
t	time [s]
t_r	time lag [s]
T	bulk temperature [K]
$\bar{T}$	mean temperature [K]
$\hat{T}$	local temperature [K]
$\vec{U}$	axial velocity [m s^{-1}]
V	volume [m^3].

Greek symbols

Δ	difference operator
φ	heat flux density [W m^{-2}]
λ	thermal conductivity [W m K^{-1}]
ρ	density [kg m^{-3}]
Σ	total surface area for heat transfer [m^2]
$\mathscr{T}$	global volumic temperature [K]
τ	time constant [s].

Subscripts

a	annular tube
c	cold
e	entry value
f	fluid
h	hot
i	inner tube
min	minimum value
max	maximum value
s	exit value
0	initial condition
∞	final condition.

with parallel flow, when the flow rates are varying simultaneously at the two inlets without using the equivalence method developed in Refs. [14] and [15].

2. MODELLING

We consider a double-pipe heat exchanger with insulated walls submitted to simultaneous sudden change of the temperatures or the flow rates at the two entrances. To study the behaviour of this exchanger in transient phase we make the following assumptions:

- the physical properties of the fluids are constant during the unsteady state;
- the heat conduction in the fluids and the pipes is neglected;
- the heat generation by viscous dissipation is neglected.

We subdivide the heat exchanger into four parts:

(1) the inner tube (i);
(2) the outer tube (a);
(3) the hot fluid (h);
(4) the cold fluid (c).

The energy balance for every part of the heat exchanger can be written as:

$$\int_V \frac{\partial(\rho C_p \hat{T})}{\partial t}\,dv + \int_S \rho C_p \hat{T}\vec{U}\vec{n}\,ds = -\int_S \vec{\varphi}\vec{n}\,ds \quad (1)$$

where V and S are, respectively, the volume and the surface of the element, $\vec{U}$ and $\hat{T}$ are the local velocity and temperature, respectively, $\vec{n}$ is the normal to the surface and $\vec{\varphi}$ is the local heat flux density. The calculation of every term in the previous equation for every part of the heat exchanger gives the set of equations:

$$\begin{cases} C_h \dfrac{\partial \mathscr{T}_h}{\partial t} + q_{th}(T_{hs} - T_{he}) = -\varphi_h \\ C_c \dfrac{\partial \mathscr{T}_c}{\partial t} + q_{tc}(T_{cs} - T_{ce}) = -\varphi_c - \varphi_a \\ C_i \dfrac{\partial \mathscr{T}_i}{\partial t} = \varphi_c + \varphi_h \\ C_a \dfrac{\partial \mathscr{T}_a}{\partial t} = \varphi_a \end{cases} \quad (2)$$

where C_j is the heat capacity, T_j is the bulk temperature and $\mathscr{T}_j$ is the mean volumic temperature ($j = h, c, i, a$) given by:

$$\mathscr{T}_i = \frac{1}{L}\int_L \bar{T}_i\,dx \quad (3)$$

where

$$\bar{T}_i = \frac{1}{S}\int_S \hat{T}_i\,dx \quad (4)$$

is the mean temperature across a section, the subscripts e and s are used for entry and exit values of the temperature, q_{th} and q_{tc} are the heat capacity flow rates of the hot and the cold fluid, respectively.

By adding equation (2) we get:

$$C\frac{\partial \mathcal{I}}{\partial t}+q_{tc}(T_{cs}-T_{ce})+q_{th}(T_{hs}-T_{he})=0 \quad (5)$$

where C and $\mathcal{I}$ are, respectively, the total heat capacity and the mean volumic temperature of the heat exchanger; they are given by these equations:

$$\begin{cases} C=C_h+C_c+C_i+C_a \\ \mathcal{I}=\dfrac{C_h\mathcal{I}_h+C_c\mathcal{I}_c+C_i\mathcal{I}_i+C_a\mathcal{I}_a}{C}. \end{cases} \quad (6)$$

In the following, we consider the case where the flow rates change and we make the assumptions that the flows are globally parallel for both co-current and counter-flow.

The experimental study of Azilinon and Pierson has shown that the exit temperature of the heat exchanger has an exponential shape after a lapse of time t_r. So, by using a first-order approximation, the solution of equation (5) can be written as:

$$\begin{cases} t\leqslant t_r: & T(t)=T_0 \\ t>t_r: & \dfrac{T(t)-T_\infty}{T_0-T_\infty}=\exp\left(-\dfrac{t-t_r}{\tau}\right) \end{cases} \quad (7)$$

with t_r, time lag; τ, time constant; T_0, initial exit temperature in steady-state; T_∞, final exit temperature in steady-state.

We observe that to obtain the response of the heat exchanger in transient phase, we must determine the two parameters (τ and t_r). Moreover, we make the assumption that the time constant and the time lag have the same values for both fluids and for the mean volumic temperature too. The expression of the first parameter can be obtained analytically, but due to the difficulty to obtain the second one analytically, it must be determined experimentally.

2.1. *The time constant*

Under the previous assumptions and using equation (7) for every temperature we can write:

$$\begin{cases} \dfrac{\mathcal{I}(t)-\mathcal{I}_\infty}{\mathcal{I}_0\pm\mathcal{I}_\infty}=\exp\left(-\dfrac{t-t_r}{\tau}\right) \\ \dfrac{T_{hs}(t)-T_{hs\infty}}{T_{hs0}-T_{hs\infty}}=\exp\left(-\dfrac{t-t_r}{\tau}\right) \\ \dfrac{\mathcal{I}_{cs}(t)-\mathcal{I}_{cs\infty}}{T_{cs0}-T_{cs\infty}}=\exp\left(-\dfrac{t-t_r}{\tau}\right) \end{cases} \quad (8)$$

which can be written as:

$$\begin{cases} T_{hs}(t)-T_{hs\infty}=\dfrac{T_{hs0}-T_{hs\infty}}{\mathcal{I}_0-\mathcal{I}_\infty}(\mathcal{I}(t)-\mathcal{I}_\infty) \\ T_{cs}(t)-T_{cs\infty}=\dfrac{T_{cs0}-T_{cs\infty}}{\mathcal{I}_0-\mathcal{I}_\infty}(\mathcal{I}(t)-\mathcal{I}_\infty) \end{cases}. \quad (9)$$

The energy conservation gives us the relation:

$$q_{th}(T_{he}-T_{hs\infty})=q_{tc}(T_{ce}-T_{cs\infty}) \quad (10)$$

so, by using equations (9) and (10), equation (5) can be written as:

$$C\frac{\partial\mathcal{I}}{\partial t}+\left(q_{th}\frac{T_{hs0}-T_{hs\infty}}{\mathcal{I}_0-\mathcal{I}_\infty}+q_{tc}\frac{T_{cs0}-T_{cs\infty}}{\mathcal{I}_0-\mathcal{I}_\infty}\right)\mathcal{I}$$

$$=\left(q_{th}\frac{T_{hs0}-T_{hs\infty}}{\mathcal{I}_0-\mathcal{I}_\infty}+q_{tc}\frac{T_{cs0}-T_{cs\infty}}{\mathcal{I}_0-\mathcal{I}_\infty}\right)\mathcal{I}_\infty. \quad (11)$$

On the other hand, $\mathcal{I}$ is the solution of the equation:

$$\tau\frac{\partial\mathcal{I}}{\partial t}+\mathcal{I}=\mathcal{I}_\infty \quad (12)$$

then we can deduce the expression of the time constant:

$$\tau=\frac{C(\mathcal{I}_0-\mathcal{I}_\infty)}{q_{th\infty}(T_{hs0}-T_{hs\infty})+q_{tc\infty}(T_{cs0}-T_{cs\infty})}. \quad (13)$$

We can observe that the time constant depends on the initial and final exit bulk temperatures which can be expressed as a function of the initial and final entrance bulk temperatures and flow rates in steady-state.

2.2. *The bulk and the mean temperatures*

In fully developed laminar flow the velocity profile is parabolic, so generally at any cross-section of the heat exchanger, the mean temperature and the bulk temperature have different expressions, but in the case of turbulent flow the temperature and velocity profiles are quite uniform, so in this case the bulk and mean temperatures are nearly identical. On the other hand, according to the flow arrangement in the heat exchanger, the exit mean temperature has different expressions for parallel or counter flow, consequently the time constant has different expressions too, but the numerical results for both cases have shown a little difference between these values. This observation is confirmed experimentally by Azilinon [11]. So, practically, it is sufficient to give the value of τ for the simplest case, the parallel flow.

2.3. *The case of turbulent flow*

In the case of parallel flows, and when the hot fluid is in the inner tube, the exit bulk temperatures in the steady-state are given by these equations:

for the exit hot temperature

$$T_{hs}=\frac{q_{th}}{q_{tc}+q_{th}}$$

$$\times\left\{T_{he}+\frac{q_{tc}}{q_{th}}T_{ce}+\frac{q_{tc}}{q_{th}}(T_{he}-T_{ce})[1-E(1+R)]\right\} \quad (14)$$

for the exit cold temperature

$$T_{cs} = \frac{q_{th}}{q_{tc}+q_{th}} \times \left\{T_{he} + \frac{q_{tc}}{q_{th}} T_{ce} - \frac{q_{tc}}{q_{th}}(T_{he}-T_{ce})[1-E(1+R)]\right\}. \quad (15)$$

The mean volumic temperatures of the hot and cold fluids are given, respectively, by:

$$\mathscr{I}_h = \frac{q_{th}}{q_{tc}+q_{th}} \left\{T_{he} + \frac{q_{tc}}{q_{th}} T_{ce} + \frac{E}{\mathrm{NTU}}\frac{q_{tc}}{q_{th}}(T_{he}-T_{ce})\right\} \quad (16)$$

and

$$\mathscr{I}_c = \frac{q_{th}}{q_{tc}+q_{th}} \left\{T_{he} + \frac{q_{tc}}{q_{th}} T_{ce} - \frac{E}{\mathrm{NTU}}(T_{he}-T_{ce})\right\}. \quad (17)$$

These of the inner and annular tubes are given, respectively, by:

$$\mathscr{I}_i = \frac{q_{th}}{q_{tc}+q_{th}} \times \left\{ T_{he} + \frac{q_{tc}}{q_{th}} T_{ce} + \frac{E}{\mathrm{NTU}} \frac{\frac{q_{tc}}{q_{th}} - \frac{h_{ic}}{h_{ih}}}{1+\frac{h_{ic}}{h_{ih}}}(T_{he}-T_{ce}) \right\} \quad (18)$$

and

$$\mathscr{I}_a = \frac{q_{th}}{q_{tc}+q_{th}} \left\{T_{ce} + \frac{q_{tc}}{q_{th}} T_{ce} - \frac{E}{\mathrm{NTU}}(T_{he}-T_{ce})\right\} \quad (19)$$

where E, R and NTU are given by:

$$E = \frac{1-\exp[-\mathrm{NTU}(1+R)]}{1+R} \quad (20)$$

$$R = \frac{q_{tmin}}{q_{tmax}} \quad (21)$$

$$\mathrm{NTU} = \frac{k\Sigma}{q_{tmin}} \quad (22)$$

$$\frac{1}{k} = \frac{1}{h_{ih}} + \frac{r_2}{\lambda_i}\mathrm{Ln}\left(\frac{r_2}{r_1}\right) + \frac{r_2}{r_1}\frac{1}{h_{ac}} \quad (23)$$

$$\Sigma = 2\pi r_2 l. \quad (24)$$

h_i and h_a are the convection heat transfer coefficients for the inner and annular tubes, respectively, and r_1 and r_2 are the radii of the inner tube. We can mention that the use of the heat transfer coefficients in unsteady state has no meaning, but as we observe in this formulation they are used only in the initial and the final steady-states which are valid.

Then the global mean volumic temperature of the heat exchanger can be written as:

$$\mathscr{I} = \frac{q_{th}}{q_{tc}+q_{th}} \left\{T_{he} + \frac{q_{tc}}{q_{th}} T_{ce} + \frac{E}{C \cdot \mathrm{NTU}}(T_{he}-T_{ce}) \times \left(\frac{q_{tc}}{q_{th}} C_h - (C_c+C_a) + \Gamma C_i\right)\right\} \quad (25)$$

where Γ is given by:

$$\Gamma = \frac{\frac{q_{tc}}{q_{th}} - \frac{h_{ic}}{h_{ih}}}{1+\frac{h_{ic}}{h_{ih}}}. \quad (26)$$

2.4. *Analytical expression of the time constant*

2.4.1. *General case.* Knowing the flow rates at the entrance of the heat exchanger:

$$t = 0, \quad q_{th} = q_{th0}, \quad q_{tc} = q_{tc0}$$

$$t \geqslant 0, \quad q_{th} = q_{th\infty}, \quad q_{tc} = q_{tc\infty}$$

and the expressions for the exit temperatures, the time constant τ can be written as:

$$\tau = \frac{C}{q_{th\infty}H_h + q_{tc\infty}H_c} \quad (27)$$

where H_h and H_c are given by:

$$H_h = \frac{(T_{hs0}-T_{hs\infty})}{(\mathscr{I}_0-\mathscr{I}_\infty)} \quad (28)$$

$$H_c = \frac{(T_{cs0}-T_{cs\infty})}{(\mathscr{I}_0-\mathscr{I}_\infty)}. \quad (29)$$

Then the expressions of H_h and H_c can be written as:

$$H_h = \frac{q_{tc\infty}E_\infty(q_{tc0}+q_{th0})(1+R_\infty) - q_{tc0}E_0(q_{tc\infty}+q_{th\infty})(1+R_0)}{q_{th0}q_{tc\infty} - q_{th\infty}q_{tc0} + q_{th0}(q_{tc\infty}+q_{th\infty}) \times \delta_0 - q_{th\infty}(q_{tc0}+q_{th0})\,\delta_\infty} \quad (30)$$

$$H_c = \frac{q_{th0}E_0(q_{tc\infty}+q_{th\infty})(1+R_0) - q_{th\infty}E_\infty(q_{tc0}+q_{th0})(1+R_\infty)}{q_{th0}q_{tc\infty} - q_{th\infty}q_{tc0} + q_{th0}(q_{tc\infty}+q_{th\infty}) \times \delta_0 - q_{th\infty}(q_{tc0}+q_{th0})\,\delta_\infty} \quad (31)$$

where

$$\delta_0 = \frac{E_0}{C \cdot \mathrm{NTU}_0}\left[\frac{q_{tc0}}{q_{th0}} C_h - C_c + \Gamma_0 C_i - C_a\right] \quad (32)$$

$$\delta_\infty = \frac{E_\infty}{C \cdot \mathrm{NTU}_\infty}\left[\frac{q_{tc\infty}}{q_{th\infty}} C_h - C_c + \Gamma_\infty C_i - C_a\right]. \quad (33)$$

2.4.2. *Special cases.*

2.4.2.1. *Flow rate step on the cold fluid*

In this case the flow rate of the hot fluid is constant ($q_{th} = q_{th0} = q_{th\infty}$), but that of the cold one varies from q_{tc0} to $q_{tc\infty}$, by using equations (30)–(33) and (27) we get:

$$\tau = \frac{C\{(q_{tc\infty}-q_{tc0}) + (q_{tc\infty}+q_{th})\,\delta_0 - (q_{tc0}+q_{th})\,\delta_\infty\}}{(q_{tc\infty}-q_{tc0})(q_{tc\infty}+q_{th})(1+R_0)E_0}. \quad (34)$$

2.4.2.2. *Flow rate step on the hot fluid*

In this case the flow rate of the cold fluid is constant ($q_{tc} = q_{tc0} = q_{tc\infty}$), but that of the hot one varies from q_{th0} to $q_{th\infty}$, by using equations (30)–(33) and (27) we get:

$$\tau = \frac{C\{q_{tc}(q_{th0}-q_{th\infty})+q_{th0}(q_{tc}+q_{th\infty}) \times \delta_0 - q_{th\infty}(q_{tc}+q_{th0})\,\delta_\infty\}}{q_{tc}(q_{th0}-q_{th\infty})(q_{tc}+q_{th\infty})(1+R_0)E_0}. \quad (35)$$

2.5. *Case of laminar flow*

In the case of laminar flow, as mentioned above, the bulk temperature and the mean temperature are different. Then the global volumic mean temperature of the heat exchanger is given in ref. [6]:

$$\mathscr{I} = \frac{q_{th}}{q_{tc}+q_{th}}\left\{T_{he}+\frac{q_{tc}}{q_{th}}T_{ce}+\frac{E}{C\,\mathrm{NTU}}(T_{he}-T_{ce}) \times\left(\frac{q_{tc}}{q_{th}}C_h-C_c-\left(\frac{1}{G}-\left(1-\frac{1}{G}\right)\Gamma\right)C_a+\Gamma C_i\right)\right\} \quad (36)$$

where G is a geometric parameter

$$G = \frac{2a_h}{(r_3^2-r_2^2)\left(b\,\mathrm{Ln}(r_3/r_2)+\dfrac{(r_3^2-r_2^2)^2}{4\,\mathrm{Ln}(r_3/r_2)}-\dfrac{3}{16}(r_3^4-r_2^4)\right)} \quad (37)$$

a_h and b are given by:

$$a_h = \frac{5}{32}\left(r_2^4r_3^2-\frac{(r_3^6+2r_2^6)}{3}\right) + \frac{b}{2}(r_3^2\,\mathrm{Ln}(r_3/r_2)-\frac{1}{2}(r_3^2-r_2^2)) - \frac{(r_3^2-r_2^2)}{16\,\mathrm{Ln}(r_3/r_2)}(r_2^4\,\mathrm{Ln}(r_3/r_2)+2r_2^2r_3^2-\tfrac{5}{4}r_3^4-\tfrac{3}{4}r_2^4) \quad (38)$$

$$b = \frac{r_3^4}{4}-\frac{r_3^2(r_3^2-r_2^2)}{4\,\mathrm{Ln}(r_3/r_2)} \quad (39)$$

r_1, r_2, r_3 and r_4 are the radii of the inner and annular tubes, respectively.

The time constant is calculated by the general formulation equations (27), (30) and (31). The expression of δ_0 and δ_∞ are:

$$\delta_0 = \frac{E_0}{C\cdot\mathrm{NTU}_0}\left[\frac{q_{tc0}}{q_{th0}}C_h-C_c+\Gamma_0C_i-\left(\frac{1}{G}-\left(1-\frac{1}{G}\right)\Gamma_0\right)C_a\right] \quad (40)$$

$$\delta_\infty = \frac{E_\infty}{C\cdot\mathrm{NTU}_0}\left[\frac{q_{tc\infty}}{q_{th\infty}}C_h-C_c+\Gamma_\infty C_i-\left(\frac{1}{G}-\left(1-\frac{1}{G}\right)\Gamma_\infty\right)C_a\right]. \quad (41)$$

2.6. *Case of laminar flow for one fluid and turbulent flow for the other*

In turbulent flow the global mean temperature of a thin annular tube can be considered equal to that of the annular fluid, but in the case of laminar flow, this equality is not valid. For this reason we have different expressions for the global mean temperature of the heat exchanger according to the type of the fluids flow in the annular tube. So, the type of the fluid flow in the annular tube imposes the formulation to be used to determine the global mean temperature of the heat exchanger, regardless of the type of the fluid flow in the inner tube, but the heat transfer coefficient in any tube is determined by the type of the fluid flow inside it. The expressions of H_h and H_c given in equations (30) and (31) are valid for both laminar and turbulent flow, but in these equations δ_0 and δ_∞ have different expressions according to the nature of the fluid flow. When the flow in the annular tube is turbulent we use equations (32) and (33), in the other case equations (40) and (41) must be used.

3. EXPERIMENTAL VERIFICATION

3.1. *Method*

The variation at the entrance of the heat exchanger can be made by two methods, either by making one step or a double step.

3.1.1. *Response to a step.* The experimental observation of the exit temperature shows that it can be approximated by an exponential curve after a lapse of time. To determine the time constant and the time lag experimentally, we make use of the energy conservation. We choose a value for the time lag, then we search for the value of the time constant that gives nearly the same area under the theoretical curve as the experimental one. As it is difficult to realise experimentally a perfect step, the determination of the exact value of the time lag and consequently that of the time constant is difficult. To overcome this problem, we can use the double step method.

3.1.2. *Response to a double step.* In this method a positive step of the flow rate is imposed on the entrance of one fluid at the instant t_0, then after a lapse of time t_a during the transient state, a negative step equal in magnitude to the positive one is imposed on the entrance of the same fluid (Fig. 1a). According to the two-parameter model, the response of the heat exchanger to the second step will appear after a lapse of time equal to t_a+t_r. As the two steps are equal in magnitude and of opposite signs, the exit temperature profile of the fluid shows an angular point at the instant t_a+t_r (Fig. 1b). This method gives better results than the first one, because the time lag can be determined more precisely and in an easier manner. So, it will be preferred to the first one to determine the time lag, but to determine the time constant, we need the maximum number of experimental points on the curve, so the first method will be used in this case.

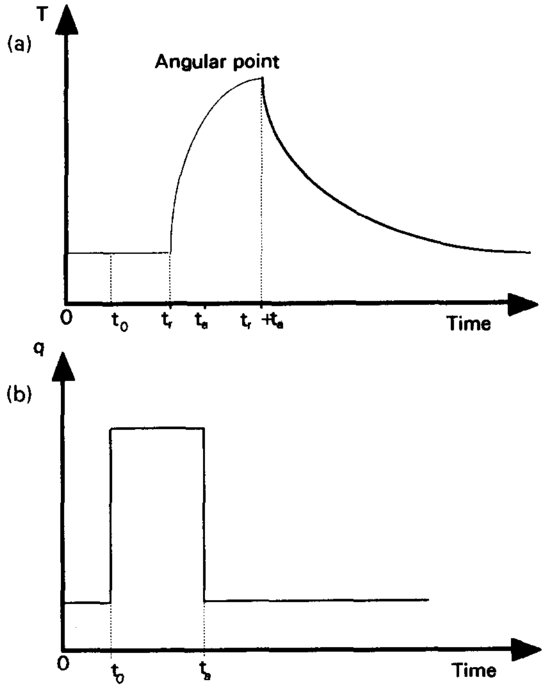

Fig. 1. (a) A double step of flow rate; (b) the exit temperature profile of the excited fluid.

3.2. *Experimental study*

Figure 2 shows the experimental installation used to validate the theoretical study. This installation can be used for many types of heat exchangers. The system of vanes allows any changes in the flow arrangement as well as the conditions at the entrance of the fluids. So, we can interchange the hot and cold fluid, and we can impose one or simultaneous temperature or flow rate steps on one or both fluids. In this study, we use water–water heat exchangers.

The double pipe heat exchanger used in this study has 2 m length. It is made of a brass inside tube of diameters 30/32 mm and a steel annular tube of diameters 50/56 mm. The theoretical and experimental results are compared for parallel flow. The cases of simultaneous and one flow rate steps are considered, as well as the position of the hot fluid in the inner or the annular tube. In the following, we present some significant results obtained among others used to validate this model.

3.2.1. *Simultaneous flow rate steps on both fluids.* In this case, the flow rates of both the hot and the cold fluids are varying from q_{h0} to $q_{h\infty}$, and from q_{c0} to $q_{c\infty}$, respectively. Table 1 shows some theoretical and experimental values of the time constant obtained in this case.

3.2.2. *One flow rate step on the hot fluid.* In this case, the flow rate of the cold fluid q_c is maintained constant but that of the hot fluid varies from q_{h0} to $q_{h\infty}$. Some examples of the obtained results are shown in Table 2.

3.2.3. *One flow rate step on the cold fluid.* In this case the flow rate of the hot fluid q_h is maintained constant but that of the cold one varies from q_{c0} to $q_{c\infty}$. Table 3 shows a sample of the time constant values obtained in this case. As we observe from Tables 1–3, the biggest difference between the theoretical and experimental results is less than 10%. Figures 3 and 4 show that the numerical and experimental exit temperature profiles are very close. So, the two-parameter model can be considered as a valid one.

4. PARAMETRIC STUDY

By examining the expression of the time constant, we observe that it depends on several dynamical and geometrical conditions, and it is difficult to group the different parameters or make them dimensionless. To analyse the effect of each parameter, a parametric study must be performed. The double pipe heat exchanger described in a previous section is used in this study. The flow is parallel with the hot fluid in the inner tube. One flow rate step is considered in the first section, then we terminate this parametric study by considering the case of simultaneous flow rate steps.

4.1. *Influence of the flow rate*

The magnitude of the initial and final flow rates for both hot and cold fluids are the most important dynamical variables. Figure 5 shows the evolution of the time constant as a function of the initial flow rate of the hot fluid for different final flow rates of the hot fluid, the flow rate of the cold fluid is maintained constant. We can observe that the time constant decreases more rapidly for little values of q_{th0} and $q_{th\infty}$ than for higher values of $q_{th\infty}$. As the values of q_{th0} increase, the time constant becomes independent of the initial flow rate of the hot fluid. On the other hand, for a fixed value of q_{th0}, the time constant decreases as the final flow rate of the hot fluid increases.

Figure 6 shows the case where the cold fluid is submitted to a flow rate step. As we observe, in this case it is difficult to give a simple law for the variation of the time constant as a function of the initial flow rate of the cold fluid, but we can realise that there is a big variation of the time constant in the transition state from laminar to turbulent flow. Moreover, this variation seems to be more sensitive as the final flow rate is in the transition state. This shows the simultaneous influence of the different parameters in such situations and the difficulty to separate one parameter from the others.

4.2. *Influence of the value of the step*

Figure 7 shows that the time constant decreases with increasing positive values of the flow rate step on the hot or the cold fluid, but it increases with increasing negative values step. This observation is globally valid regardless of the initial value of the flow rate of the fluid that will be submitted to the step.

4.3. *Influence of the length of the heat exchanger*

From Fig. 8, we observe that the time constant varies in a linear manner with the length of the

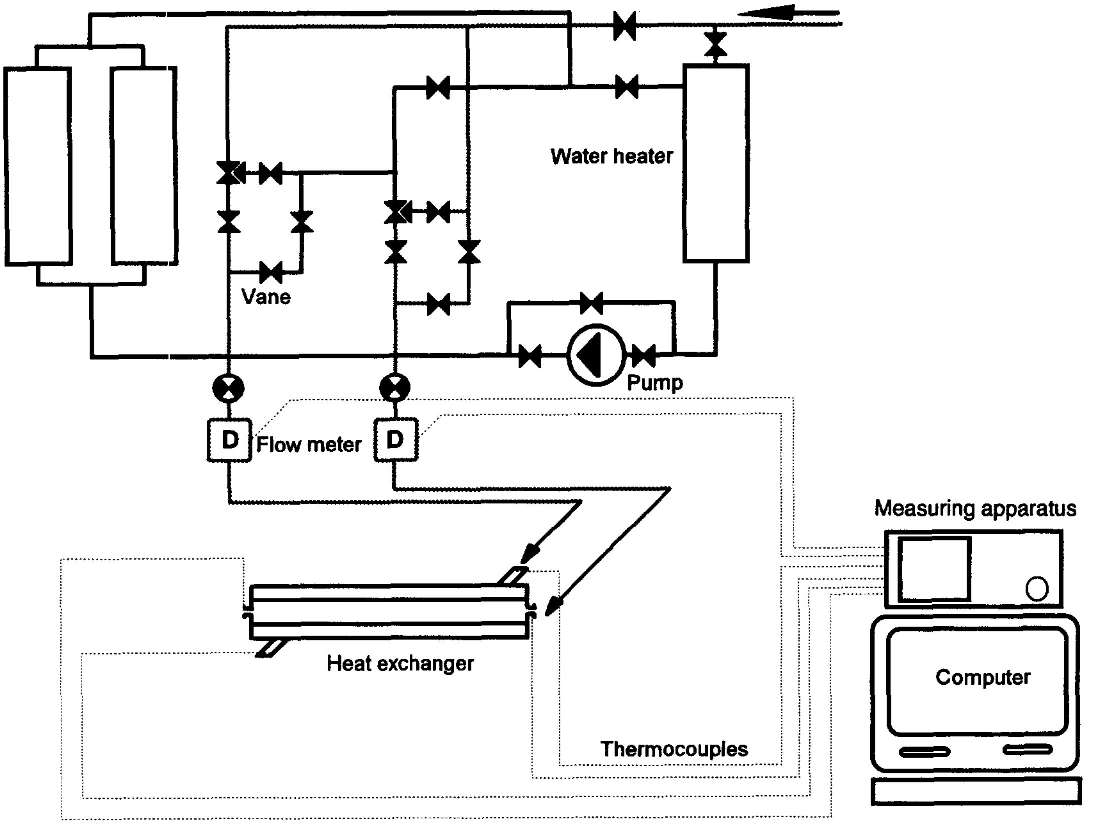

Fig. 2. Experimental installation.

Table 1. Comparison of the theoretical and experimental values of the time constant in the case of simultaneous flow rate

	Hot fluid				Cold fluid			Time constant		
	Flow rate [kg s^{-1}]		Reynolds		Flow rate [kg s^{-1}]		Reynolds			
	Initial	Final	number		Initial	Final	number	τ_{exp}	τ_{the}	$\Delta\tau/\tau$
Position	q_{h0}	$q_{h\infty}$	$Re_{h\infty}$	Position	q_{c0}	$q_{c\infty}$	$Re_{c\infty}$	(s)	(s)	%
Inner	0.2	0.08	6925	Annular	0.13	0.27	1905.6	7.9	8.8	10.2
Annular	0.26	0.16	2533.6	Inner	0.17	0.27	10418	7.7	8	3.9
Annular	0.26	0.16	2533.6	Inner	0.27	0.17	6559	4	4.3	7.5

Table 2. Comparison of the theoretical and experimental values of the time constant in the case of one flow rate step on the hot fluid

	Hot fluid				Cold fluid		Time constant		
	Flow rate [kg s^{-1}]		Reynolds			Reynolds			
	Initial	Final	number		Flow rate [kg s^{-1}]	number	τ_{exp}	τ_{the}	$\Delta\tau/\tau$
Position	q_{h0}	$q_{h\infty}$	$Re_{h\infty}$	Position	q_c	$Re_{c\infty}$	(s)	(s)	%
Inner	0.05	0.18	15582	Annular	0.32	2258.5	5.3	5.3	0
Annular	0.08	0.25	3958.7	Inner	0.17	6559	7	7.2	2.8
Annular	0.09	0.18	2850.3	Inner	0.14	5401.6	6.2	5.8	6.5

exchanger regardless of which fluid is submitted to the flow rate step. This type of variation is not evident by examining the theoretical formulation of the time constant, because the length of the heat exchanger influences several parameters at the same time.

4.4. *Influence of the heat capacity of the inner tube*

In order to study the influence of the heat capacity of the inner tube, we have changed its thickness. This variation is made by maintaining the outer diameter constant and varying the inner one, consequently the

Table 3. Comparison of the theoretical and experimental values of the time constant in the case of one flow rate step on the cold fluid

Hot fluid			Cold fluid				Time constant		
Position	Flow rate [kg s^{-1}] q_h	Reynolds number $Re_{h\infty}$	Position	Flow rate [kg s^{-1}] Initial q_{c0}	Final $q_{c\infty}$	Reynolds number $Re_{c\infty}$	τ_{exp} (s)	τ_{the} (s)	$\Delta\tau/\tau$ %
Inner	0.13	11 253.7	Annular	0.05	0.15	3086.6	12.4	13.5	8.9
Annular	0.25	3958.7	Inner	0.27	0.15	5787.5	7.9	7.4	6.7
Annular	0.2	3167	Inner	0.27	0.08	3086.6	9.7	9.6	1

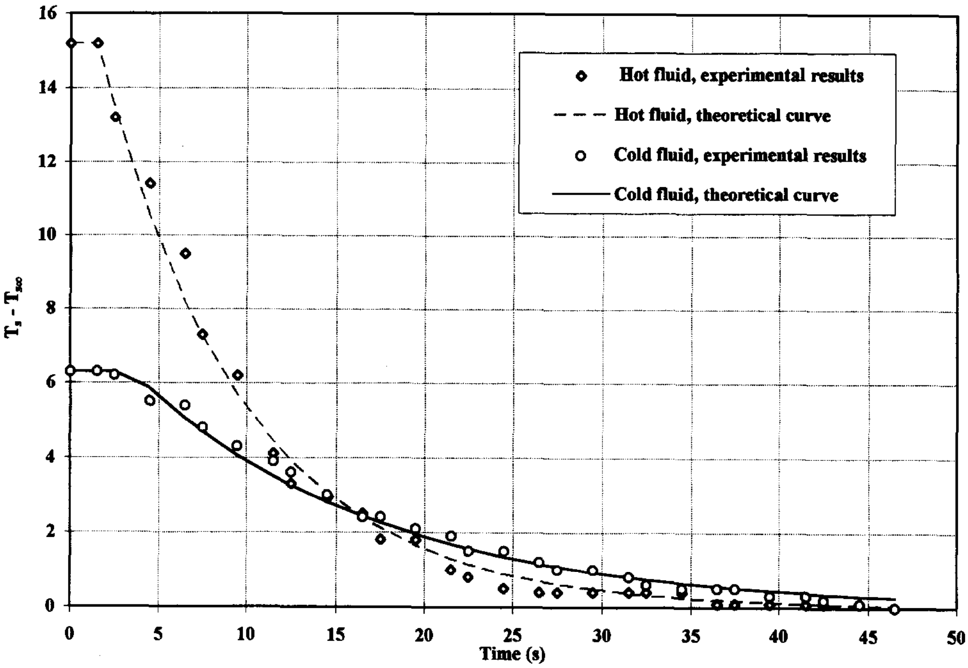

Fig. 3. Evolution of the exit temperature for the hot and cold fluid, case of simultaneous flow rate steps on both fluids, the hot fluid in the annular tube, $q_{h0} = 0.20$ kg s^{-1}, $q_{h\infty} = 0.10$ kg s^{-1}, $q_{c0} = 0.05$ kg s^{-1} and $q_{c\infty} = 0.10$ kg s^{-1}.

characteristics of the inner fluid flow will change. Figure 9 shows that the time constant is proportional to the thickness of the inner tube in any flow arrangement. So, to increase velocity of the response of the heat exchanger to any variation in the inlet, the thickness of the inner tube must be as thin as possible, which seems natural.

4.5. *Case of simultaneous flow rate steps*

To figure out the response of the heat exchanger in different configurations, we complete this parametric study by considering the case of simultaneous flow rate steps. Figures 10 and 11 show that the time constant evolves in the same manner as in the case where only the hot fluid is submitted to a flow rate step.

5. SHELL AND TUBE HEAT EXCHANGER

The two-parameter model can be extended to the case of shell and tube heat exchanger. In order to attain this objective, the equivalence method is used by many authors like Azilinon *et al.* [14] and Henrion and Feidt [15]. The aim of this method is to determine the dimensions and the thermophysical properties of a double pipe heat exchanger that will behave in the same manner as the shell and tube heat exchanger. To use this method, some geometrical and physical conditions must be satisfied. For this reason, the authors suppose an equal heat transfer coefficient in the annular tube or an equal global heat transfer coefficient for the two heat exchangers. This method is simple, but the hypothesis made previously may be difficult to realise and verify.

The formulation of the two parameter model for the double pipe heat exchanger can be used in the case of the shell and tube heat exchanger in turbulent and globally parallel flow without the need of the equivalence method. In this case, we must pay attention to the use of the correct expression for the heat transfer coefficient.

The flow in the annular tube is generally turbulent even for low values of Reynolds number, because of

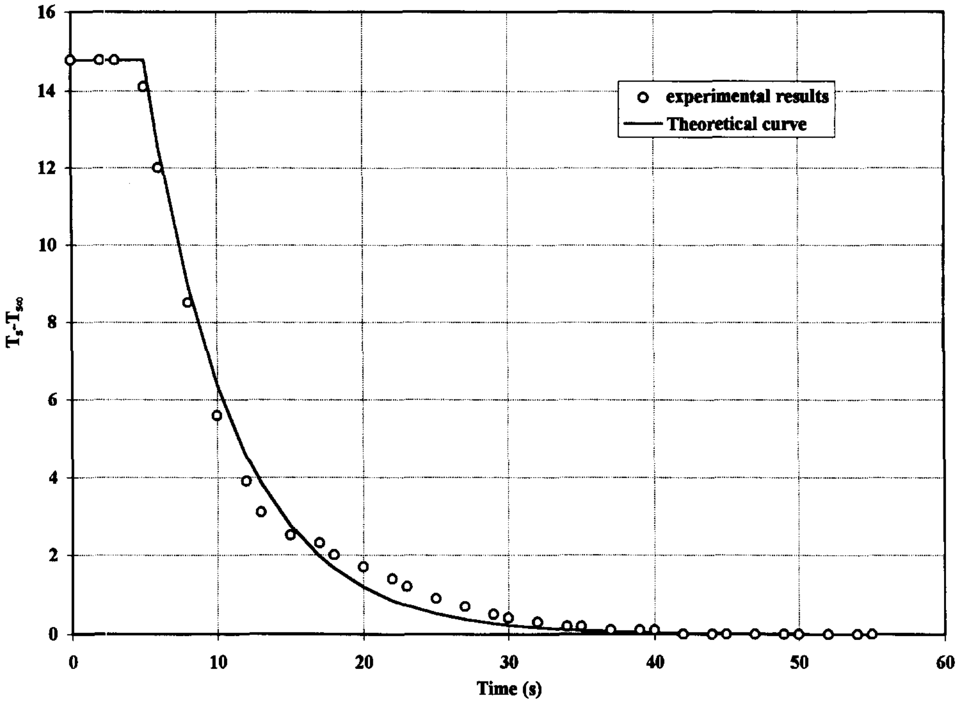

Fig. 4. Evolution of the exit temperature of the hot fluid, one flow rate step on the hot fluid in the inner tube, $q_{h0} = 0.03$ kg s^{-1}, $q_{h\infty} = 0.23$ kg s^{-1} and $q_c = 0.25$ kg s^{-1}.

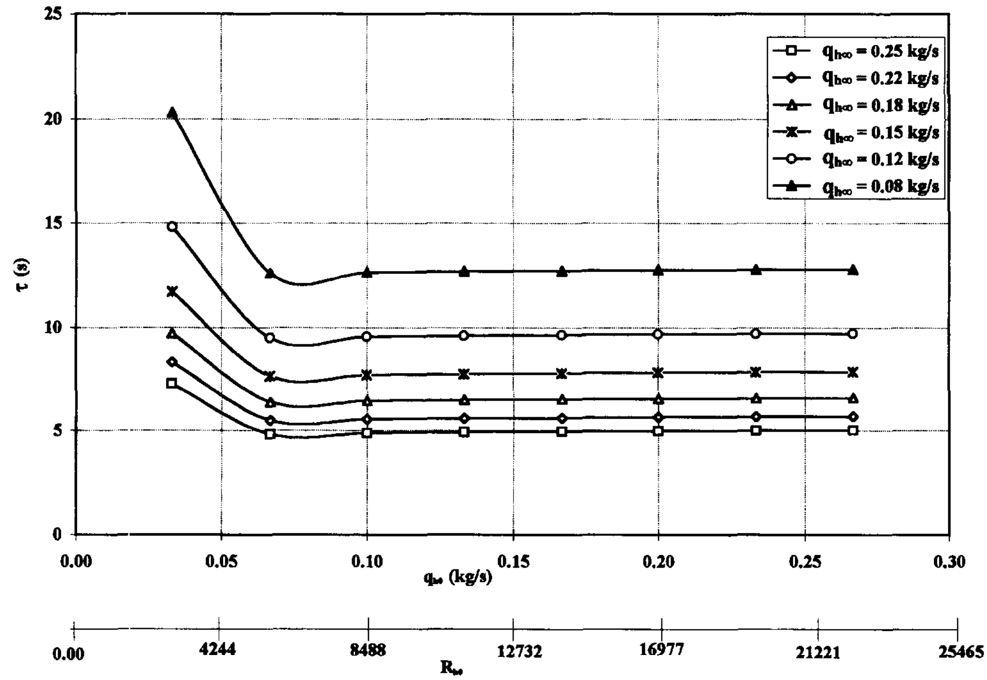

Fig. 5. Influence of the initial flow rate of the hot fluid in the inner tube q_{h0}, for different values of the final flow rate, $q_{c\infty} = 0.12$ kg s^{-1}.

the use of a great number of inner tubes and baffles which mix the fluid and make its flow turbulent, but in the inner tubes, the flow may be laminar or turbulent according to the flow rate. So, as mentioned previously in Section 2.6, the formulation for turbulent flow must be used. The heat transfer coefficients for the inner and annular tubes may be calculated from classical correlation in the literature [16]. The number

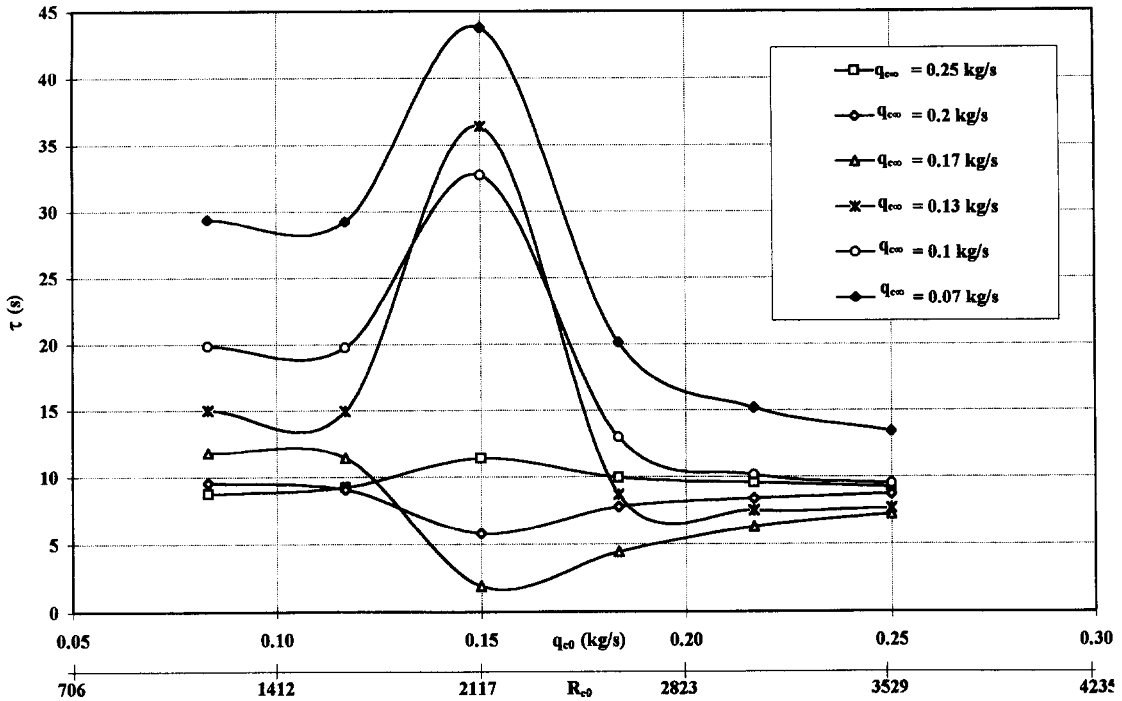

Fig. 6. Influence of the initial flow rate of the cold fluid in the annular tube, for different values of the final flow rate, $q_{h\infty} = 0.12$ kg s^{-1}.

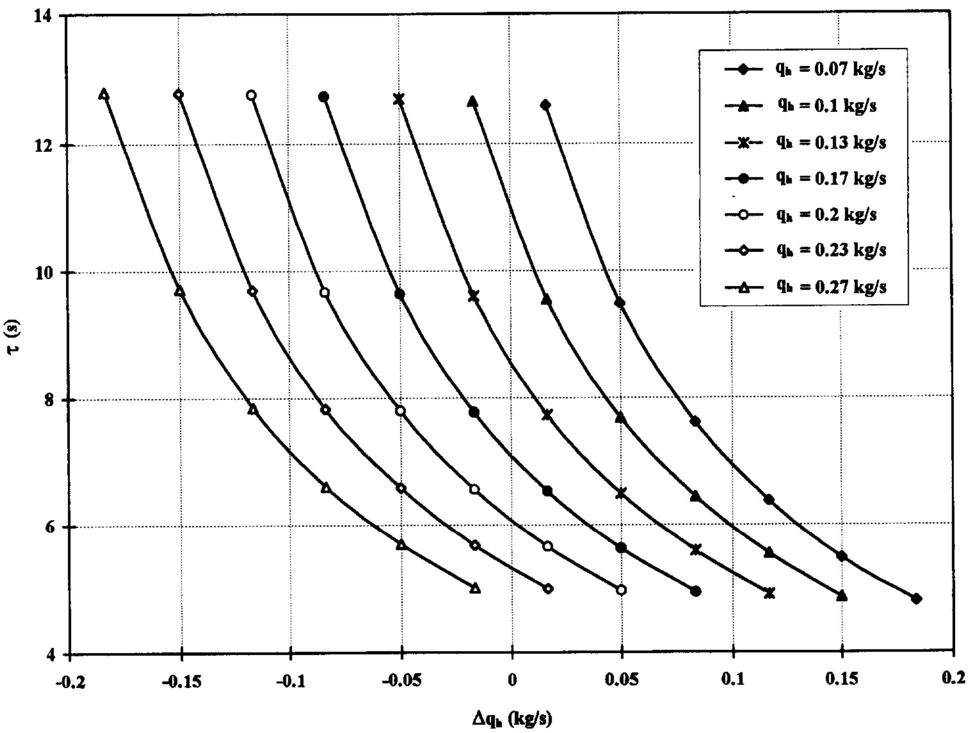

Fig. 7. Influence of the magnitude of the flow rate step, for different values of q_{h0}, $q_c = 0.12$ kg s^{-1}, the hot fluid in the inner tube.

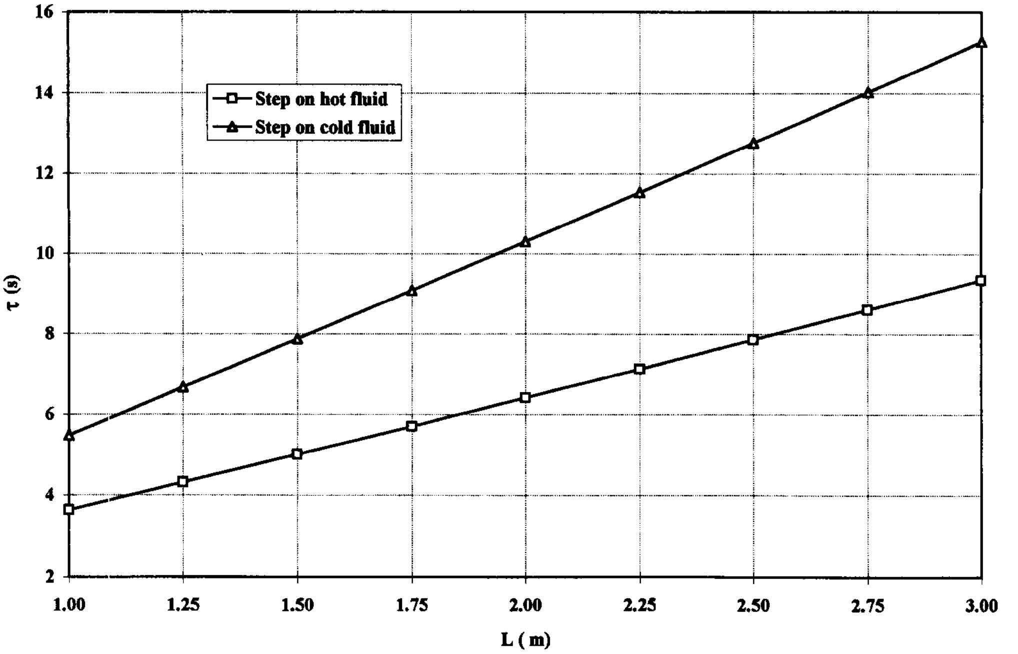

Fig. 8. Variation of τ as a function of the length of the heat exchanger for two cases: (1) flow rate step on the hot fluid $q_{h0} = 0.08$ kg s^{-1}, $q_{h\infty} = 0.18$ kg s^{-1} and $q_c = 0.12$ kg s^{-1}; (2) flow rate step on the cold fluid $q_{c0} = 0.08$ kg s^{-1}, $q_{c\infty} = 0.18$ kg s^{-1} and $q_h = 0.12$ kg s^{-1}.

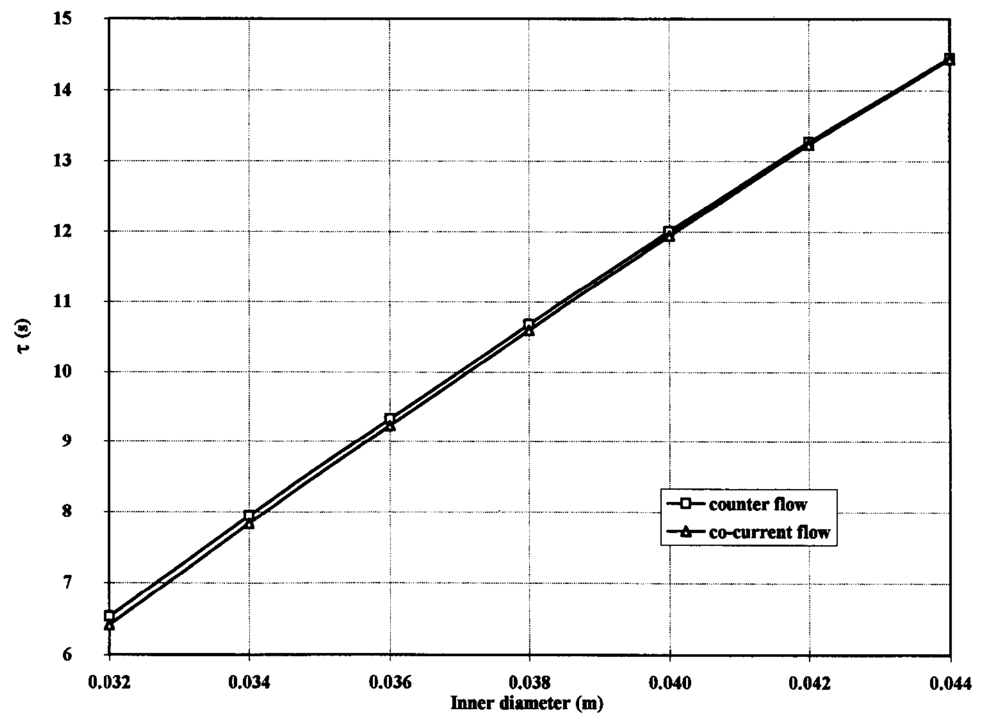

Fig. 9. Influence of the thickness of the inner tube, case of one flow rate step on the hot fluid in the inner tube $q_{h0} = 349$ W K^{-1}, $q_{th\infty} = 767$ W K^{-1} and $q_{tc} = 488$ W K^{-1}.

of transfer units NTU is calculated by equations (22) where the global heat transfer coefficient k is determined from equation (23). The effectiveness E is given by equation (20) in which the unbalance factor R is determined by equation (21).

To validate this method, a comparison between the theoretical and experimental results for the time constant is shown in the following. The shell and tube heat exchanger used in this study has a length of 950 mm. It is made of a steel annular tube of 212.5/215.6 mm and 38 copper inner tubes of diameters 12/14 mm. The inner tubes have a triangular disposition and are

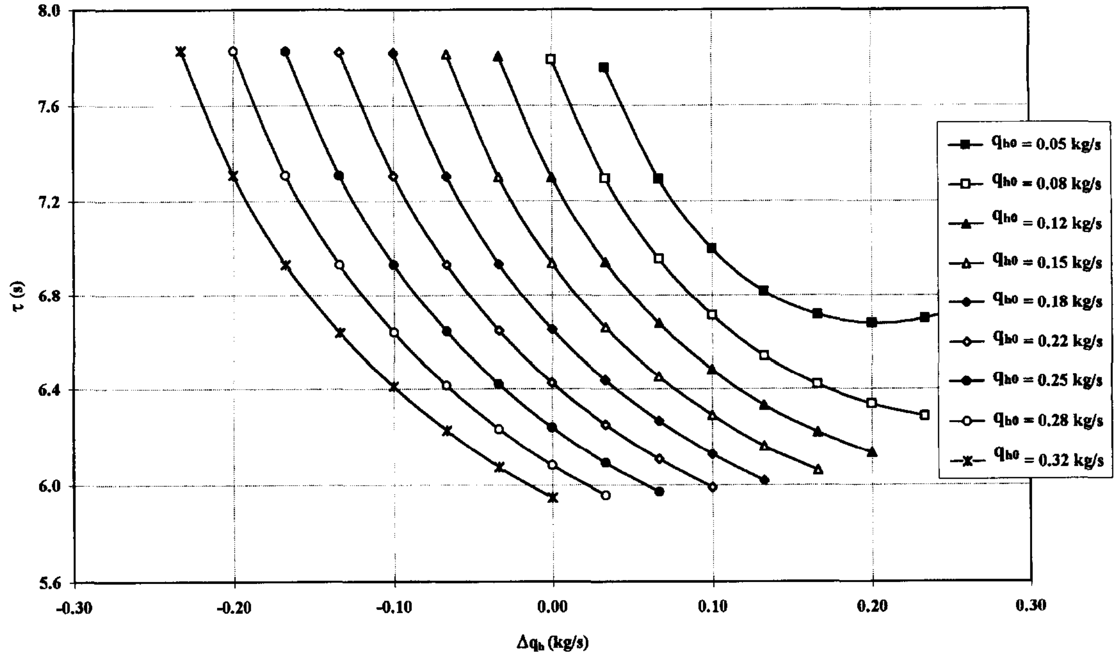

Fig. 10. Influence of the value of the step, for different values of the initial flow rate of the hot fluid in the inner tube, the cold flow rate varies from $q_{c0} = 0.02 \text{ kg s}^{-1}$ to $q_{c\infty} = 0.32 \text{ kg s}^{-1}$.

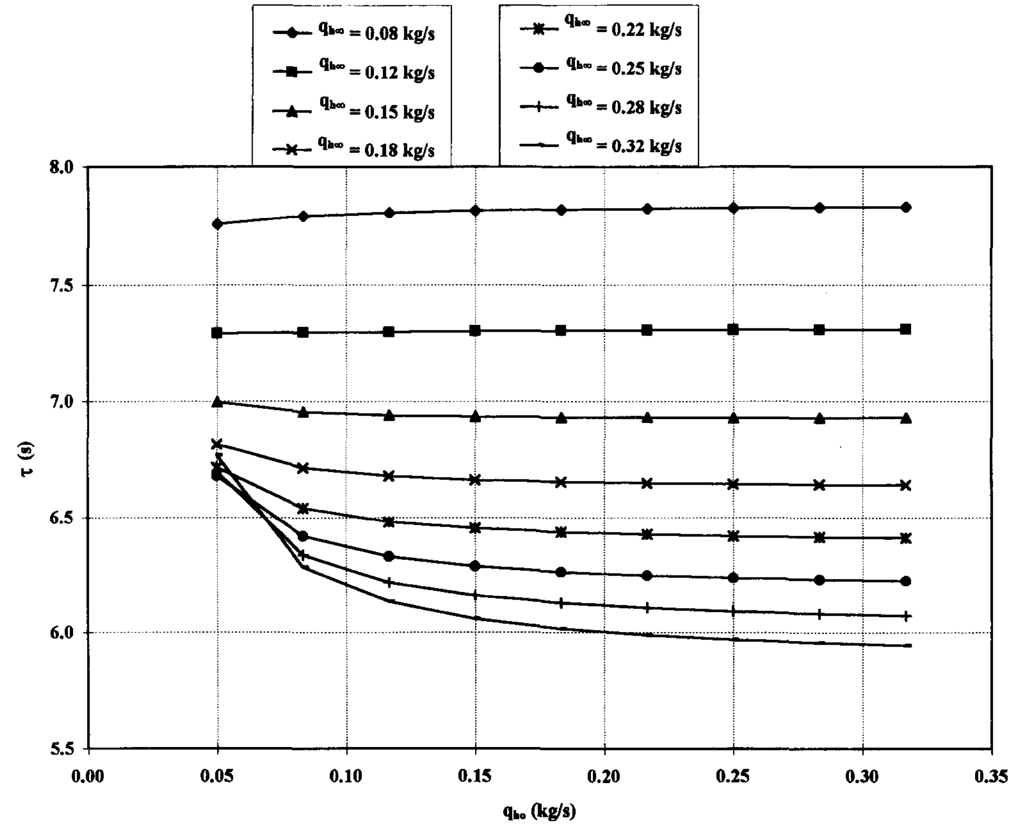

Fig. 11. Influence of the initial flow rate of the hot fluid in the inner tube for different values of final flow rate, $q_{c0} = 0.02 \text{ kg s}^{-1}$, $q_{c\infty} = 0.32 \text{ kg s}^{-1}$.

Table 4. Comparison of the theoretical and experimental values of the time constant in the case of one flow rate step on the hot fluid (case of shell and tube heat exchanger)

Cold fluid	Hot fluid flow rate [kg s^{-1}]		Inlet temperature (°C)		Outlet temperature (°C)		Time constant		
q_{tc0}	q_{th0}	$q_{th\infty}$	T_{eh}	T_{ec}	T_{sh}	T_{sc}	τ_{exp} (s)	τ_{the} (s)	$\Delta\tau/\tau$ (%)
0.15	0.08	0.12	36.7	22.6	41.5	25.8	35.67	38.37	7
0.17	0.16	0.23	43.3	29.4	46.5	30.5	40	37.04	7.4
0.18	0.07	0.16	29.8	18.5	40.3	24.8	36.48	36.9	1.1
0.18	0.14	0.22	38.2	22.9	45.3	28.6	45.78	42.87	6.3
0.24	0.23	0.18	43.7	22.4	41.7	21.8	36.5	35.2	3.5
0.27	0.28	0.33	43.2	23.4	44.7	24.7	33.43	30	10
0.38	0.25	0.36	36.3	17.9	38.9	19.6	13.84	15.37	9.9

maintained by seven baffles. Table 4 shows some examples of the different results obtained to validate this model in the case of shell and tube heat exchanger. We can observe that this method gives acceptable results. The biggest difference between experimental and theoretical results is less than 10%. So, the two parameter model gives the same precision for both the shell and tube and double pipe heat exchangers.

6. CONCLUSION

This study demonstrates that the two parameter model is a valid method to determine the response of double pipe heat exchangers, and can be extended to shell and tube heat exchangers without making use of the equivalence method. In both cases, the difference between the theoretical and experimental results are less than 10%. An analytical expression of the time constant is available. As the number of parameters included in its expression is high, it is difficult to find general rules to describe the response of the heat exchanger in any case. To have the exact response of the heat exchanger, a study case by case must be performed. From the parametric study, we can observe that the time constant decreases as the flow rate or the flow rate step increases, so the response of the heat exchanger is more rapid.

To complete the model, a study of the time lag is needed. Finally, knowing the response of the heat exchanger to a step of flow rate, it is possible to determine the response of the heat exchanger to any variation of the flow rate at the entrance.

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